CHAPTER 1

FUNDAMENTAL CONCEPTS AND PRINCIPLES

1–1 INTRODUCTION TO MECHANICS

Mechanics is the branch of physical science that deals with the state of rest or motion of bodies under the action of forces. Mechanics is the foundation for most engineering sciences and it is an indispensable prerequisite to most engineering or technical courses. The principles of mechanics are used in almost all technical analysis and design. A thorough understanding of these principles and their applications are of paramount importance for students in the engineering and technical programs.

Mechanics is divided into three branches: statics, dynamics, and strength of materials. *Statics* concerns the equilibrium of bodies under the action of *balanced forces. Dynamics* deals with the *motions* of bodies under the action of *unbalanced forces. Strength of materials* deals with the relationships among the external forces applied to the bodies, the resulting stresses (intensity of internal forces), and deformation (change of size or shape). The determination of the proper sizes of structural members to satisfy strength and deformation requirements are also important topics of strength of materials.

In the study of statics and dynamics, all bodies are assumed to be perfectly rigid. A *rigid body* is a solid in which the distance between any two points in the body remain unchanged. This is an idealization. In reality, deformations do occur in all bodies when they are subjected to forces. However, the deformations are usually very small and they can be neglected in the statics and dynamic analyses, without appreciable errors.

In the study of the strength of materials, deformation of structural members becomes very important because the concerns are the strength and stiffness of structural or machine members. Strength and stiffness are directly or indirectly related to the deformation, even if the deformation is very small.

This book concentrates on two major topics. The first eight chapters deal with statics, and the remaining twelve chapters deal with strength of materials. Dynamics will not be covered in this book.

1–2
THE NATURE OF A FORCE

A *force* is any effect that may change the state of rest or motion of a body. It represents the action of one body on another. The existence of a force can be observed by the effects that the force produces. Force is applied either by direct physical contact between bodies or by remote action. Gravitational, electrical, and magnetic forces are applied through remote action. Most other forces are applied by direct contact.

Forces Exerted by Direct Contact. The forces exerted on a rope pulled by someone's hands and the forces between a beam and its supports are examples of forces applied by direct contact. Less obvious cases of contact forces occur when a solid body comes in contact with a liquid or a gas. For example, forces exist between water and the hull of a boat, and similarly, between air and airplane wings.

Forces Exerted Through Remote Action. When a ball is thrown into the air, it falls to the ground. The pull of the earth's gravity, exerted through remote action, causes the ball to fall. The attraction force of the earth is usually referred to as the *weight* of the body. A satellite is kept in its orbit around the earth by the gravitational attraction from the earth. When a magnet attracts a small piece of iron through remote action, the magnetic force causes the iron to move without any physical contact.

Characteristics of a Force. A force can be defined completely by a magnitude, a direction, and a point of application. These are called the *characteristics of a force.* The magnitude of a force is described by a number with a proper unit. The direction of a force is indicated by a line (called the *line of action*) with an arrowhead. The *point of application* is the point at which the force is exerted. For example, the force applied to the block in Fig. 1–1 is described as a 100-lb (magnitude) force acting along a horizontal line (line of action) to the right (direction) through point *A* (point of application).

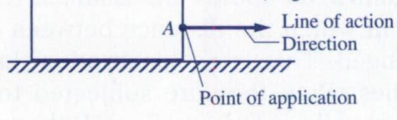


FIGURE 1–1

1–3
SCALAR AND VECTOR QUANTITIES

The quantities that we deal with in mechanics can be classified into two groups.

Scalar Quantities. Scalar quantities can be described completely by a magnitude. Examples of scalar quantities are length, area, volume, speed, mass, time, etc.

Vector Quantities. A vector quantity is characterized by its magnitude, direction, line of action, and sometimes point of application. Furthermore, vector quantities must be added by the *parallelogram law* (to be discussed in the next chapter); that is, vectors must be added geometrically rather than algebraically. As described in the preceding section, a force is a vector quantity. Other examples of vector quantities include moment, displacement, velocity, acceleration, etc.

1–4 TYPES OF FORCES

Forces can be classified into the following types.

Distributed and Concentrated Forces. A *distributed force* is exerted on a line, over an area, or throughout an entire volume. The weight of a beam can be treated as a distributed force over its length. A *concentrated force* is an idealization in which a force is assumed to act at a point. A force can be regarded as a concentrated force if the area of application is relatively small compared to the total surface area of the body.

External and Internal Forces. A force is called an *external force* if it is exerted on the body by another body. If a structure is formed by several connected components, the forces holding the component parts together are *internal forces* within the structure. In Fig. 1–2, the applied forces **P** and **Q**, together with the reactions A_x, A_y, and B_y at the supports, are external forces to the truss. The internal forces are developed in the truss members due to the applied loads and the reactions. These internal forces are responsible for holding the truss together.

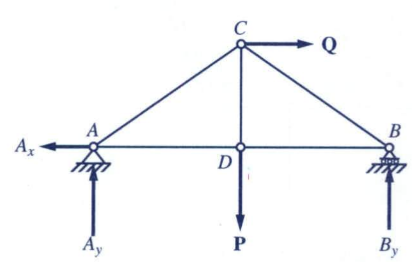


FIGURE 1–2

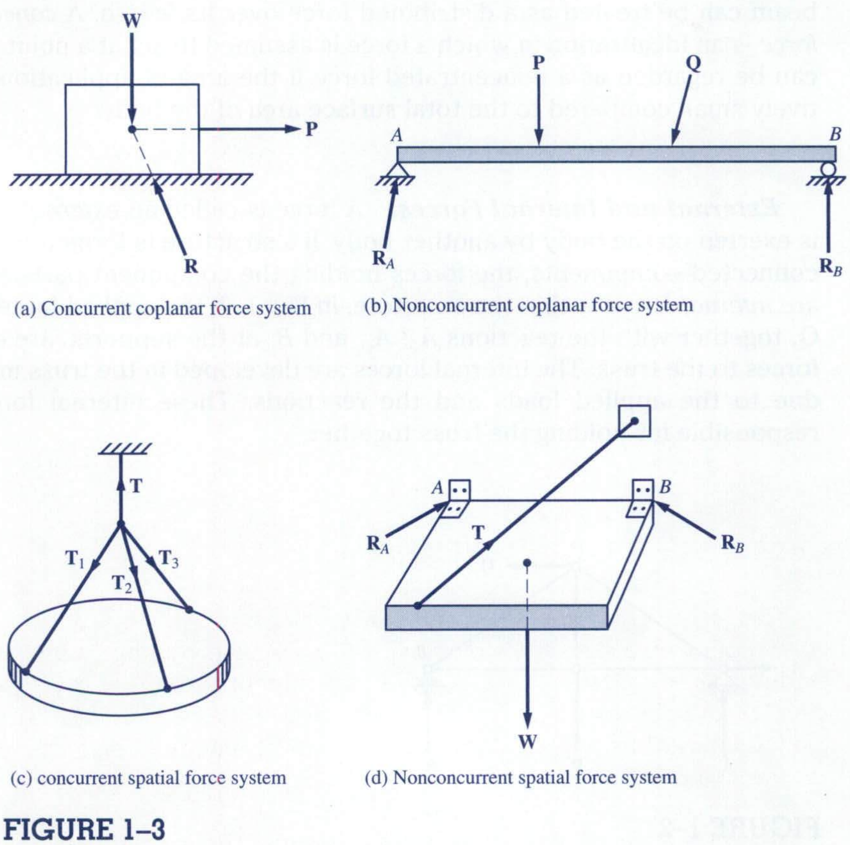
1–5
TYPES OF FORCE SYSTEMS

Forces treated as a group constitute a force system. Forces whose lines of action lie in the same plane are called *coplanar forces.* Forces whose lines of action act in a three-dimensional space are called *spatial forces.* If the lines of action of all the forces in a system pass through a common point, they are said to be *concurrent.* On the other hand, if there is no common point of intersection, the forces are said to be *nonconcurrent.* Depending on whether the forces are coplanar or spatial, concurrent or nonconcurrent, force systems may be classified into the following types.

Concurrent Coplanar Force System. The lines of action of all the forces in the system pass through a common point and lie in the same plane, as shown in Fig. 1–3a.

Nonconcurrent Coplanar Force System. The lines of action of all the forces in the system lie in the same plane but do not pass through a common point, as shown in Fig. 1–3b.

Spatial Force System. The lines of action of all the forces in the system do not lie in the same plane. Spatial force systems can be either concurrent (Fig. 1–3c) or nonconcurrent (Fig. 1–3d).



(a) Concurrent coplanar force system (b) Nonconcurrent coplanar force system

(c) concurrent spatial force system (d) Nonconcurrent spatial force system

FIGURE 1–3

1–6 NEWTON'S LAWS

In the latter part of the seventeenth century, Sir Isaac Newton (1642–1727) formulated three laws governing the equilibrium and motion of a particle (a point mass). *Newton's laws* now form the foundation of *Newtonian mechanics.*

First Law. *A particle remains at rest or continues to move along a straight line with a constant velocity if the force acting on it is zero.*

Second Law. *If the force acting on a particle is not zero, the particle accelerates (changes velocity with respect to time) in the direction of the force, and the magnitude of the acceleration (the rate of change of velocity per unit time) is proportional to the magnitude of the force.*

Third Law. *The forces of action and reaction between interactive bodies always have the same magnitudes and opposite directions.*

The first law deals with the condition for equilibrium of a particle. It provides the foundation for the study of *statics.* The second law is the basis for the study of *dynamics* and it may be expressed in the vector equation as:

$$\mathbf{F} = m\mathbf{a} \tag{1–1}$$

where $\mathbf{F}$ = the force acting on the particle

m = the mass of the particle

$\mathbf{a}$ = the acceleration of the particle caused by the force

The third law is important for both statics and dynamics. It states that active and reactive forces always exist in equal and opposite pairs. For example, the downward weight of a body resting on a table is accompanied by an upward reaction of the same magnitude exerted by the table on the body. The third law applies equally well to forces exerted through remote action. For instance, two magnets always attract each other with equal and opposite forces.

1–7 THE PRINCIPLE OF TRANSMISSIBILITY

The *principle of transmissibility* states that the point of application of a force acting on a rigid body may be placed anywhere along its line of action without altering the conditions of equilibrium or motion of the rigid body. As an example, consider the disabled car shown in Fig. 1–4a. The car is pulled forward by a force **F** applied to the front bumper. Using the principle of transmissibility, the pulling force **F** may be replaced by an equivalent force **F′** acting on the rear bumper (Fig. 1–4b). We see that the state of motion of the car is unaffected, and all the other external forces, **W**, $\mathbf{R}_1$, and

$\mathbf{R}_2$, acting on the car remain unchanged whether the car is pulled by **F** or pushed by **F′**, as long as the two forces have the same magnitude, have the same direction, and act along the same line.

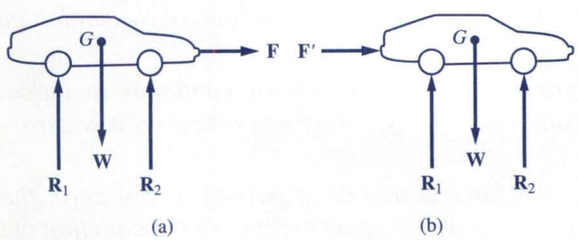


FIGURE 1–4

One must be aware, however, that the internal effect of a force on a body is dependent on its point of application. Hence, the principle of transmissibility does not apply if our concern is the internal force or deformation. For example, the force **F** applied at point *B* of the bracket in Fig. 1–5a cannot be placed at point *C* (Fig. 1–5b) if our concern is the internal force or the deflection in the part labeled *BC*.

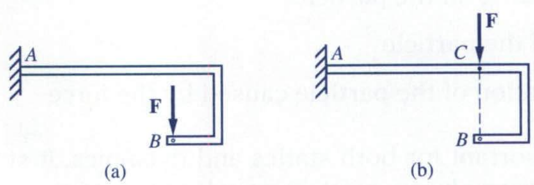


FIGURE 1–5

1–8 SYSTEMS OF UNITS

Equation 1–1 involves the measurements of length, time, mass, and force. Hence, the units of these quantities must satisfy Equation 1–1 and they cannot be chosen independently. If three *base units* are chosen, the fourth unit must be derived from Equation 1–1 in terms of the base units.

Currently, two systems of units are used in engineering practices in the United States: the *U.S. customary system of units* (the old English system) and the *International System* or *SI units* (from the French *Système International d'Unités*). The SI units have now been widely adopted throughout the world. In the United States, the U.S. customary units are gradually being replaced by the SI units. During the transition years, however, engineers must be familiar with both systems.

U.S. Customary Units. The three base units in this system are:

Length: foot (ft)
Force: pound (lb)
Time: second (s)

The base unit pound is dependent on the gravitational attraction of the earth, which varies with location; hence, this system is also referred to as a *gravitational system.*

The unit of mass, called the *slug,* is a derived unit. From Equation 1–1, we have:

$$m = F/a$$

$$1 \text{ slug} = (1 \text{ lb})/(1 \text{ ft/s}^2) = 1 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Thus, the slug is a derived unit for mass expressed in the base units as $\text{lb} \cdot \text{s}^2/\text{ft}$. The slug is commonly used in dynamics. In this book, it is rarely used.

The weight of a body (due to the gravitational attraction) causes the body to accelerate at 32.2 ft/s^2 on the surface of the earth. This quantity is usually denoted by g and is called the *gravitational acceleration.* Equation 1–1 may be rewritten as:

$$W = mg \tag{1–2}$$

From this equation, the weight of a 1-slug mass on the surface of the earth is:

$$\begin{aligned} W &= mg \\ &= (1 \text{ slug})(32.2 \text{ ft/s}^2) \\ &= (1 \text{ lb} \cdot \text{s}^2/\text{ ft})(32.2 \text{ ft/s}^2) \\ &= 32.2 \text{ lb} \end{aligned}$$

Other U.S. customary units frequently encountered in engineering practice are:

$$\begin{aligned} 1 \text{ mile (mi)} &= 5280 \text{ ft} \\ 1 \text{ inch (in.)} &= \frac{1}{12} \text{ ft} \\ 1 \text{ kilo-pound (kip)} &= 1000 \text{ lb} \\ 1 \text{ U.S. ton (ton)} &= 2000 \text{ lb} \\ 1 \text{ minute (min)} &= 60 \text{ s} \\ 1 \text{ hour (h)} &= 60 \text{ min} = 3600 \text{ s} \end{aligned}$$

SI Units. The three base SI units are:

Length: meter (m)
Mass: kilogram (kg)
Time: second (s)

The SI units are an *absolute system* because the three base units chosen are independent of the location where the measurements are made.

The unit of force, called the *newton* (N), is a derived unit expressed in terms of the three base units. From Equation 1–1:

$$F = ma$$

$$1\ \text{N} = (1\ \text{kg}) \cdot (\text{m/s}^2) = 1\ \text{kg} \cdot \text{m/s}^2$$

Thus, the unit of force, newton, is equivalent to $\text{kg} \cdot \text{m/s}^2$. With gravitational acceleration, $g = 9.81\ \text{m/s}^2$, the weight of a 1-kg mass on the surface of the earth is:

$$\begin{aligned} W &= mg \\ &= (1\ \text{kg})(9.81\ \text{m/s}^2) \\ &= 9.81\ \text{kg} \cdot \text{m/s}^2 = 9.81\ \text{N} \end{aligned}$$

When a quantity is either very large or very small, the units may be modified by using a *prefix.* Commonly used SI prefixes are shown in Table 1–1. The following are some typical examples of the use of prefixes in mechanics:

$$1\ \text{kg} = 1000\ \text{g}$$

$$1\ \text{km} = 10^3\ \text{m} = 1000\ \text{m}$$

$$1\ \text{m} = 1000\ \text{mm}$$

$$1\ \text{kN} = 10^3\ \text{N} = 1000\ \text{N}$$

$$1\ \text{Mg (metric ton)} = 10^3\ \text{kg} = 1000\ \text{kg}$$

Unit reductions with quantities involving prefixes can be done simply by moving the decimal point three places to the right (multiply by 10^3) or to the left (multiply by 10^{-3}). For example:

TABLE 1–1 Commonly Used SI Prefixes

Prefix	*Symbol*	*Multiplication Factor*
giga-	G	10^9 = 1 000 000 000[a]
mega-	M	10^6 = 1 000 000
kilo-	k	10^3 = 1 000
centi- [b]	c	10^{-2} = 0.01
milli-	m	10^{-3} = 0.001
micro-	μ	10^{-6} = 0.000 001
nano-	n	10^{-9} = 0.000 000 001

[a] Use a space rather than a comma to separate numbers in groups of three, counting from the decimal point in both directions. Spaces may be omitted for numbers of four digits. Commas are not used for this purpose because they are used as decimal points in some countries.

[b] The use of *centi* should be avoided in general except for certain measurements of areas and volumes.

$$1.59 \text{ km} = 1590 \text{ m} = 1.59 \times 10^3 \text{ m}$$
$$4.83 \text{ Mg} = 4830 \text{ kg} = 4.83 \times 10^3 \text{ kg}$$
$$75.4 \text{ mm} = 0.0754 \text{ m} = 75.4 \times 10^{-3} \text{ m}$$

With the exception of the base unit kg, in general a prefix must be used as the leading symbol in a derived unit. For example, a spring constant (force per unit length of deformation) can be expressed as kN/m, but not as N/mm. A moment (force multiplied by distance) can be expressed as kN · m, but not as N · km or N · mm.

1-9 UNIT CONVERSION

Changing units within a system is call *unit reduction.* Changing units from one system to another system is called *unit conversion.* In this book, problems are solved in the same system of units as in the given data. Therefore, unit conversion is not needed in problem solutions. In actual engineering applications, however, unit conversions are sometimes necessary. The following conversion factors are useful:

$$1 \text{ ft} = 0.3048 \text{ m}$$
$$1 \text{ slug} = 14.59 \text{ kg}$$
$$1 \text{ lb} = 4.448 \text{ N}$$

For a more extensive list of conversion factors, refer to Table 1–2, p. 12, which lists the U.S. customary units for quantities frequently used in mechanics and their SI equivalents.

EXAMPLE 1–1

Convert a velocity of 30 mph into its equivalent value in m/s.

Solution. The unit mile must be reduced to feet first before being converted to meters. Thus:

$$30 \text{ mph} = \left(\frac{30 \cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{5280 \cancel{\text{ft}}}{1 \cancel{\text{mi}}}\right)\left(\frac{0.3048 \text{ m}}{1 \cancel{\text{ft}}}\right)\left(\frac{1 \cancel{\text{h}}}{3600 \text{ s}}\right)$$
$$= 13.41 \text{ m/s} \qquad \Leftarrow \textbf{Ans.}$$

The conversion factors used above are each equal to unity because the numerator and denominator of each factor represent the same measurement. The value of a quantity is not altered if it is multiplied by factors of unity.

Using the conversion factor for mph and m/s listed in Table 1–2 (1 mph = 0.4470 m/s), the result can be obtained more conveniently as

$$30 \text{ mph} = (30 \cancel{\text{mi/h}})\left(\frac{0.4470 \text{ m/s}}{1 \cancel{\text{mi/h}}}\right)$$
$$= 13.41 \text{ m/s}$$

EXAMPLE 1–2

Convert a moment 10 N · m to an equivalent value in the appropriate U.S. customary units.

Solution. The given units are force multiplied by length. Thus, the corresponding U.S. customary units are lb · ft:

$$10\ \text{N}\cdot\text{m} = (10\ \text{N}\cdot\text{m})\left(\frac{1\ \text{lb}}{4.448\ \text{N}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right)$$

$$= 7.38\ \text{lb}\cdot\text{ft} \qquad \Leftarrow \textbf{Ans.}$$

Using the conversion factor for moment, N · m and lb · ft, listed in Table 1–2 (1 lb · ft = 1.356 N · m), we get

$$10\ \text{N}\cdot\text{m} = (10\ \text{N}\cdot\text{m})\left(\frac{1\ \text{lb}\cdot\text{ft}}{1.356\ \text{N}\cdot\text{m}}\right)$$

$$= 7.38\ \text{lb}\cdot\text{ft}$$

TABLE 1–2 U.S. Customary Units and Their SI Equivalents

Quantity	*U.S. Customary Unit*	*SI Equivalent*
Length	ft	0.3048 m
	in.	25.40 mm
	mi	1.609 km
Mass	slug	14.59 kg
Force	lb	4.448 N
	kip	4.448 kN
Area	ft^2	0.0929 m^2
	$in.^2$	$0.6452 \times 10^{-3}\ m^2$
Volume	ft^3	0.02832 m^3
	$in.^3$	$16.39 \times 10^{-6}\ m^3$
Velocity	ft/s	0.3048 m/s
	mi/h (mph)	0.4470 m/s
	mi/h (mph)	1.609 km/h
Acceleration	ft/s^2	0.3048 m/s^2
Moment of a force	lb · ft	1.356 N · m
	lb · in.	0.1130 N · m
Pressure or stress	lb/ft^2 (psf)	47.88 Pa (pascal or N/m^2)
	$lb/in.^2$ (psi)	6.895 kPa (kN/m^2)
Spring constant	lb/ft	14.59 N/m
	lb/in.	175.1 N/m
Load intensity	lb/ft	14.59 N/m
	kip/ft	14.59 kN/m
Area moment of inertia	$in.^4$	$0.4162 \times 10^{-6}\ m^4$
Work or energy	lb · ft	1.356 J (joule or N · m)
Power	lb · ft/s	1.356 W (watt or N · m/s)
	hp (1 horsepower = 550 ft · lb/s)	745.7 W (watt or N · m/s)

1–10 CONSISTENCY OF UNITS IN AN EQUATION

An equation relating several physical quantities is called a physical equation. Every term in a physical equation must be *dimensionally homogeneous*; that is, each term must be reduced to the same units.

When substituting the values of known quantities into an equation, it is extremely important that the units in the quantities be consistent. *Students should form the habit of carrying units with all quantities when substituting into an equation and making sure that the result is in the correct units.* The following examples illustrate the proper procedure.

EXAMPLE 1–3

Using the equation $s = vt$, find the distance s in feet traveled by a car at a constant speed $v = 60$ mi/h for a period of $t = 30$ s.

Solution. Before the speed v and the period t are substituted in $s = vt$, we must first reduce the units of v from mi/h to ft/s. Thus:

$$v = \left(\frac{65\ \cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{5280\ \text{ft}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$
$$= 95.33\ \text{ft/s}$$

$$s = vt$$
$$= \left(95.33\ \frac{\text{ft}}{\cancel{\text{s}}}\right)(30\ \cancel{\text{s}})$$
$$= 2860\ \text{ft}$$ ⇐ **Ans.**

EXAMPLE 1–4

Solve the equation $mv = Ft$ for the velocity v and find its value if the force $F = 2.56$ N, the time $t = 16.8$ s, and the mass $m = 1.89$ kg.

Solution. Dividing both sides of the equation by m, we get:

$$v = \frac{Ft}{m}$$

Substituting the given quantities into the right-hand side of the equation, we get:

$$v = \frac{Ft}{m}$$
$$= \frac{(2.56\ \text{N})(16.8\ \text{s})}{(1.89\ \text{kg})}$$

At first glance, it seems that the units lead nowhere. But we note that the unit newton (N) is a derived unit that can be expressed in the base units as $\text{kg} \cdot \text{m/s}^2$. Thus, we write:

$$v = \frac{(2.56\ \cancel{kg} \cdot m/s^{\cancel{2}})(16.8\ \cancel{s})}{(1.89\ \cancel{kg})}$$

$$= 22.8\ m/s \qquad \Leftarrow \textbf{Ans.}$$

Note that as long as consistent units (kg, m, s, and N) are used, the result for v is in the units of m/s.

EXAMPLE 1–5

The elongation e of a rod subjected to an axial load F can be computed from the equation

$$e = \frac{FL}{AE}$$

where F = axial load

L = length of the rod

A = cross-sectional area of the rod

E = modulus of elasticity of the material

Find the elongation of a steel rod subjected to an axial load of 15 kips. The rod has a length of 4 ft 6 in., and a diameter of 1 in., and the modulus of elasticity of steel is $E = 30 \times 10^6$ lb/in.^{2}.

Solution. For this problem, it is convenient to use lb and in. units. Before substituting the quantities into the equation, we must reduce each quantity to the lb and in. units:

$$F = 15\ \text{kips} = (15\ \cancel{\text{kips}})\left(\frac{1000\ \text{lb}}{1\ \cancel{\text{kip}}}\right) = 15\ 000\ \text{lb}$$

$$L = 4\ \text{ft}\ 6\ \text{in.} = (4.5\ \cancel{\text{ft}})\left(\frac{12\ \text{in.}}{1\ \cancel{\text{ft}}}\right) = 54.0\ \text{in.}$$

$$A = \frac{\pi}{4}(1\ \text{in.})^2 = 0.754\ \text{in.}^2$$

$$E = 30 \times 10^6\ \text{lb/in.}^2$$

Now these quantities, together with their units, are substituted into the equation. The reason for including the units in the substitution is to make sure that all the units are consistent and that the result is dimensionally correct.

$$e = \frac{FL}{AE}$$

$$= \frac{(15\ 000\ \cancel{\text{lb}})(54.0\ \text{in.})}{(0.785\ \cancel{\text{in.}^2})(30 \times 10^6\ \cancel{\text{lb}}/\cancel{\text{in.}^2})}$$

$$= 0.0344\ \text{in.} \qquad \Leftarrow \textbf{Ans.}$$

1–11
RULES FOR NUMERICAL COMPUTATIONS

Approximate Numbers. Although some numbers that we encounter in engineering computations are exact numbers, most numbers are approximate. *Exact numbers* are either derived from definition or obtained by counting. For example, one hour has exactly 60 minutes by definition. One inch is defined to equal 25.4 mm exactly. An automobile has four wheels by counting. *Approximate numbers* are usually obtained through some kind of measurement. For example, the distance between two points on the ground is measured to be 235.7 ft; the voltage of a house current is measured to be 115 volts.

Approximate numbers are usually written with a decimal and often include zeros that serve as placeholders. For example, the zeros in the numbers 7400 and 0.0057 are used as placeholders. The zeros in the numbers 2005 and 0.708 indicate that the values at those digits are zero, so they are not used merely as placeholders.

Significant Digits. Except for the zeros used as placeholders, all the other digits in an approximate number are considered *significant digits.* For example, the numbers 176, 0.587, 1350, 3050, 0.00408 are of three significant digits each.

Accuracy and Precision. The *accuracy* of a number refers to its number of significant digits. For example, the numbers 1570, 60.9, and 0.0805 are all accurate to three significant digits. The *precision* of a number refers to the decimal position of the last significant digit. For example, the number 1.35 is precise to the nearest hundredths (two decimal places), and the number 0.745 is precise to the nearest thousandths (three decimal places).

EXAMPLE 1–6

Determine the number of significant digits in the following groups of numbers:

(a) 1240, 254 000, 0.348, 0.005 86
(b) 304.3, 28.06, 7003, 1.704
(c) 22.40, 2.890, 8.000, 4.560×10^8

Solution

(a) The numbers 1240, 254 000, 0.348, and 0.005 86 each have three significant digits. Note that we count only the nonzero digits as significant. All the zeros in these numbers are used as placeholders.

(b) The numbers 304.3, 28.06, 7003, and 1.704 each have four significant digits. Note that the zeros in these numbers are significant because they are not used here as placeholders. They are used to indicate that, in those digits, the values are zero.

(c) The numbers 22.40, 2.890, 8.000, and 4.560×10^8 each have four significant digits. The zeros at the end of each number after the decimal point are used to show that these digits are indeed zero. These zeros should not be written unless they are significant. Note that the number 34 000 has only two significant digits. If the number is known to have four significant digits, it should be written in scientific notation* as 3.400×10^4.

* The *scientific notation* of a number is expressed as a product of a number between 1 and 10 and an integer power of 10. For example, the numbers 43 200 and 0.000 068 3 can be expressed in scientific notation as 4.32×10^4 and 6.83×10^{-5}, respectively.

EXAMPLE 1–7

A steel plate 1.25 in. thick is coated with a thin layer of paint 0.014 in. thick. Of these two values of thickness, which one has a greater accuracy and which one has a greater precision?

Solution. The number 1.25 has three significant digits, while the number 0.014 has only two significant digits. Therefore, the thickness of the plate, 1.25 in., has a greater accuracy. On the other hand, the number 1.25 is precise to the nearest hundredths, and the number 0.014 is precise to the nearest thousandths; therefore, the thickness of the paint has a greater precision.

Rules for Numerical Computation. When calculations are performed on approximate numbers, the results must be expressed with the proper number of digits. It is a common mistake to express the final result in a greater accuracy or precision than is warranted by the given data. Therefore, it is essential that the following rules are observed.

Rule 1 When approximate numbers are *multiplied or divided,* the result is expressed with the *same accuracy as the least accurate number.*

Rule 2 When approximate numbers are *added or subtracted,* the result is expressed with the *same precision as the least precise number.*

EXAMPLE 1–8

Calculate the area of a circle from the following expression:

$$\frac{\pi (2.683)^2}{4}$$

Solution. The number 4 in the expression is an exact number, so it does not limit the accuracy of the result. The number π is a built-in feature in a calculator and accurate to at least 8 significant digits. Thus, the number 2.683, with four significant digits, is the least accurate number in the expression. Therefore, from rule 1, the final result should be expressed in four significant digits. Thus,

$$\frac{\pi\,(2.683)^2}{4} = 5.654 \qquad \Leftarrow \textbf{Ans.}$$

which is rounded off from 5.6536.

EXAMPLE 1–9

Find the sum of the following approximate numbers:

$$12.36 + 26.53 + 4.782 + 1.203 + 204.5$$

Solution. The least precise number in the expression is 204.5, which is precise to the tenths. According to rule 2, the sum should be rounded off to the tenths also. Thus,

$$\text{The sum} = 249.4 \qquad \Leftarrow \textbf{Ans.}$$

which is rounded off from 249.375.

EXAMPLE 1–10

Evaluate the following expression:

$$\frac{(27.83)(4.756)}{9.843 - 2.78}$$

Solution. After all the numbers are entered, the calculator displays a result of 18.7398. To see how many significant digits should be expressed in the result, we first note that in the denominator, the difference of the two numbers must be precise to the hundredths because the least precise number (2.78) is precise to the hundredths (rule 2). That number gives the denominator an accuracy of three significant digits and makes it the least accurate number. Then, according to rule 1, the final result should be accurate to three significant digits. Thus:

$$\text{The result} = 18.7 \qquad \Leftarrow \textbf{Ans.}$$

which is rounded off from 18.74.

The following rule of thumb is commonly used when the given data are of unknown accuracy: *Round off a result to four significant digits if its first significant digit is 1; round off a result to three significant digits if its first significant digit is other than 1.* For example, a result of 136.25 N is rounded off to 136.3 N, and a result of 2864 lb is rounded off to 2860 lb. However, all intermediate computations must be carried through with one more significant digit than in the final results. When working on a calculator, retain all digits and round only the final answer. Unless otherwise indicated, the data given in a problem is assumed to be accurate to three or four significant digits. For example, a force of 120 lb is to be read as 120.0 lb, and a length of 5.6 m is to be read as 5.60 m.

1–12
A BRIEF REVIEW OF MATHEMATICS

Mathematics Used in Mechanics. This book requires a knowledge of only basic mathematics, including arithmetic, algebra, geometry, and trigonometry. This section is designed to give students a brief review of some important mathematical skills useful in the study of mechanics.

Right Triangles. A right triangle is a closed, three-sided figure that has a right angle (angle equals 90°). The side opposite the right angle is called the *hypotenuse.* Figure 1–6 shows a right triangle, with the right angle at *C.* The sides opposite to angles *A, B,* and *C* are denoted by *a, b,* and *c*, respectively. Side *c* is the hypotenuse. With respect to angle *A, a* is the *opposite side* and *b* is the *adjacent side.* (With respect to angle *B, a* is the adjacent side and *b* is the opposite side.)

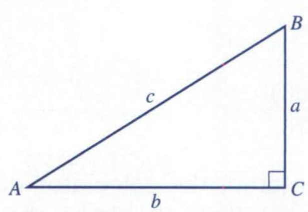


FIGURE 1–6

Since the sum of the three interior angles of a triangle is 180° and $C = 90°$, we have:

$$A + B = 90° \qquad (1\text{–}3)$$

The Pythagorean theorem. *The hypotenuse of a right triangle squared is equal to the sum of the other two sides squared*; that is:

$$c^2 = a^2 + b^2 \qquad (1\text{–}4)$$

Trigonometry of Right Triangles. Trigonometry relates the lengths of the sides of a right triangle by means of trigonometric functions. The three trigonometric functions used in this book are the sine, cosine, and tangent functions. These functions are commonly abbreviated as sin, cos, and tan, respectively. The three trigonometric functions for angle *A* are as follows:

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \tag{1–5}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c} \tag{1–6}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} \tag{1–7}$$

Since only the ratio of sides is necessary to define the trigonometric functions, the values of trigonometric functions are constant for a given angle, regardless of the size of the triangle. The trigonometric functions of an angle can be obtained by using a scientific calculator.

If two sides of a right triangle or one side and an *acute angle* (angle less than 90°) of a right triangle are known, the other unknown elements can be determined by using the Pythagorean theorem and the trigonometric functions. These processes are illustrated in the following examples.

EXAMPLE 1–11

A 16-ft ladder leans against a wall, forming an angle of 75° with the floor (see Fig. E1–11). Determine the height h that the ladder reaches on the wall.

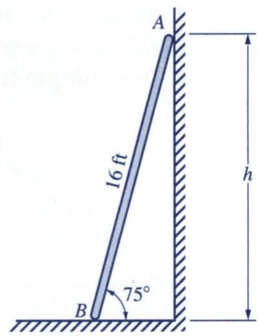


FIGURE E1–11

Solution. Using the definition of the sine function, we write:

$$\sin 75° = \frac{h}{16 \text{ ft}}$$

From which

$$h = (16 \text{ ft}) \sin 75° = 15.45 \text{ ft}$$

⇐ **Ans.**

EXAMPLE 1–12

A symmetrical roof has the dimensions indicated in Fig. E1–12(1). Determine the height h and the angle of inclination A of the roof.

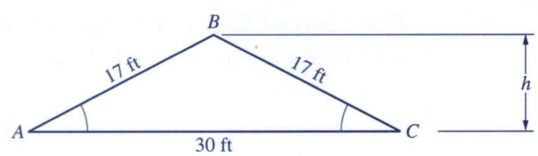


FIGURE E1–12(1)

Solution. From B draw a line BD perpendicular to AC. [see Fig. E1–12(2)]. Line BD bisects AC; thus, $AD = AC/2 = 15$ ft. Triangle ADB is a right triangle. By the Pythagorean theorem:

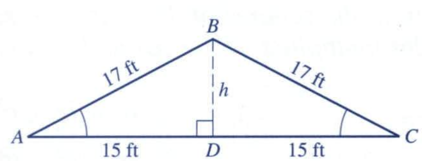


FIGURE E1–12(2)

$$h^2 = 17^2 - 15^2 = 64$$

$$h = 8 \text{ ft}$$ ⇐ **Ans.**

By the definition of the cosine function:

$$\cos A = \frac{15}{17}$$

Thus,

$$A = \cos^{-1}\left(\frac{15}{17}\right)$$

$$= 28.1°$$ ⇐ **Ans.**

where $\cos^{-1}(15/17)$ stands for the inverse cosine function of the ratio 15/17.

Oblique Triangles. An *oblique triangle* is a triangle in which none of the interior angles is equal to 90°. Figure 1–7a shows an oblique triangle with three acute angles. Figure 1–7b shows an oblique triangle with an *obtuse angle B*, which is greater than 90°. In both cases, *a*, *b*, and *c* represent the three sides of the oblique triangle opposite to the angles *A*, *B*, and *C*, respectively.

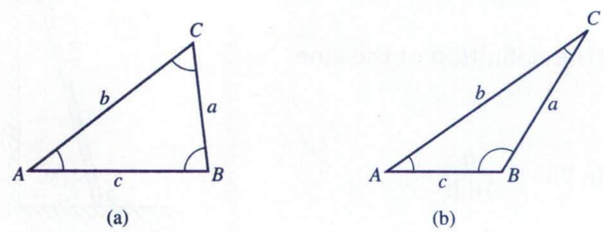


FIGURE 1–7

The sum of the three interior angles in a triangle is 180°; that is:

$$A + B + C = 180° \quad (1\text{–}8)$$

The Law of Sines. *The ratio of any side of a triangle to the sine function of its opposite angle is a constant:*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (1\text{–}9)$$

The Law of Cosines. *The square of any side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of the two sides multiplied by the cosine function of the angle between them:*

$$a^2 = b^2 + c^2 - 2bc \cos A \quad (1\text{–}10a)$$

$$b^2 = c^2 + a^2 - 2ca \cos B \quad (1\text{–}10b)$$

$$c^2 = a^2 + b^2 - 2ab \cos C \quad (1\text{–}10c)$$

The following examples demonstrate the use of these laws in solving problems that involve oblique triangles.

EXAMPLE 1–13

A 20-ft ladder AB leaning on a wall makes an angle of 25° with the wall, as shown in Fig. E1–13. The ground is 15° from the horizontal. Determine the height h that the ladder reaches on the wall.

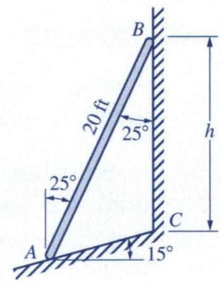


FIGURE E1–13

Solution. In triangle ABC, the interior angle at B is 25°. The other two interior angles at A and C are:

$$A = 90° - 25° - 15°$$
$$= 50°$$
$$C = 180° - 25° - 50°$$
$$= 105°$$

Applying the law of sines to the triangle ABC, we write:

$$\frac{h}{\sin 50°} = \frac{20 \text{ ft}}{\sin 105°}$$

From which we get:

$$h = \frac{(20 \text{ ft}) \sin 50°}{\sin 105°}$$
$$= 15.86 \text{ ft}$$ ⇐ **Ans.**

EXAMPLE 1–14

For the bracket support shown in Fig. E1–14, rod AB is 500 mm long and makes an angle of 60° with the vertical. The vertical distance between the supports is 600 mm. Compute the length of rod BC.

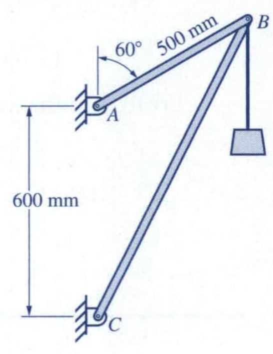


FIGURE E1–14

Solution. In the oblique triangle ABC, the interior angle at A is

$$A = 180° - 60°$$
$$= 120°$$

Now the oblique triangle ABC has two known sides and a known angle between the two sides. Applying the law of cosines to the triangle we write

$$(BC)^2 = (500)^2 + (600)^2 - 2(500)(600) \cos 120°$$

From which, we obtain:

$$BC = 954 \text{ mm} \qquad \Leftarrow \textbf{Ans.}$$

EXAMPLE 1–15

A lamp is suspended between two walls with cables AB and AC, as shown in Fig. E1–15(1). Compute the angle α between the two cables.

Solution. Refer to Fig. E1–15(2). To find the distance BC, we connect two points, B and C, and draw line CD perpendicular to the wall. Triangle BCD is a right triangle, as shown in Fig. E1–15(2). From the Pythagorean theorem, we get:

$$BC = \sqrt{(0.5 \text{ m})^2 + (2.5 \text{ m})^2}$$
$$= 2.55 \text{ m}$$

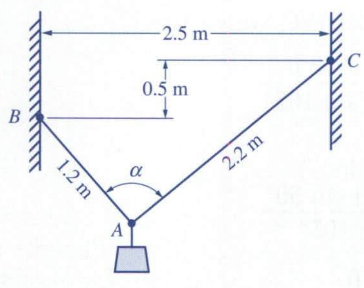


FIGURE E1–15(1)

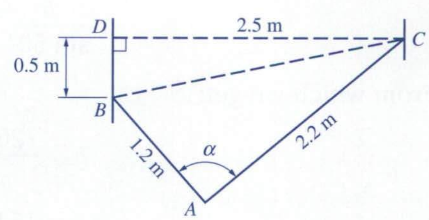


FIGURE E1–15(2)

The oblique triangle ABC has three known sides. Applying the law of cosines to the triangle, we write:

$$(2.55 \text{ m})^2 = (1.2 \text{ m})^2 + (2.2 \text{ m})^2 - 2(1.2 \text{ m})(2.2 \text{ m}) \cos \alpha$$

From which, we get

$$\cos \alpha = -0.04214$$
$$\alpha = \cos^{-1}(-0.04214)$$
$$= 92.4° \qquad \Leftarrow \textbf{Ans.}$$

Simultaneous Equations. In solving two-dimensional equilibrium problems, sometimes we need to solve two simultaneous linear equations containing two unknowns. Solving three simultaneous linear equations containing three unknowns occurs in the solutions of three-dimensional equilibrium problems. The following examples illustrate the solution of two and three simultaneous linear equations by the methods of substitution and addition or subtraction.

EXAMPLE 1–16

In solving a two-dimensional equilibrium problem, the following equations are obtained:

$$P \cos 20^\circ - Q \cos 40^\circ = 0 \quad \text{(a)}$$

$$P \sin 20^\circ + Q \sin 40^\circ = 100 \text{ lb} \quad \text{(b)}$$

where P and Q are two unknown forces. Solve the equations for the two unknowns.

Solution. Two methods are presented in the following.

Method 1: Elimination by Substitution. Solving Equation (a) for Q, we get:

$$Q = P \cos 20^\circ / \cos 40^\circ = 1.227P \quad \text{(c)}$$

Substituting into Equation (b) gives:

$$P(\sin 20^\circ + 1.227 \sin 40^\circ) = 100 \text{ lb}$$

$$P = 88.5 \text{ lb} \quad \Leftarrow \textbf{Ans.}$$

Substituting into Equation (c), we obtain:

$$Q = 108.5 \text{ lb} \quad \Leftarrow \textbf{Ans.}$$

Method 2: Elimination by Addition and Subtraction. The unknown Q can be eliminated by adding the two equations if the coefficients of Q are reduced to the same value but opposite in sign. We can reduce the coefficient of Q in Equation (a) to -1 if *every term* of the equation is divided by cos 40°:

$$\text{(a)} \div \cos 40^\circ\text{:} \quad P \cos 20^\circ / \cos 40^\circ - Q = 0$$

$$1.227P - Q = 0 \quad \text{(d)}$$

Similarly, the coefficient of Q in (b) becomes 1 if *every term* of the equation is divided by sin 40°:

$$\text{(b)} \div \sin 40^\circ\text{:} \quad P \sin 20^\circ / \sin 40^\circ + Q = (100 \text{ lb}) / \sin 40^\circ$$

$$0.5321P + Q = 155.6 \text{ lb} \quad \text{(e)}$$

If we add Equations (d) and (e), Q can be eliminated:

$$\text{(d)} + \text{(e)}\text{:} \quad 1.759P = 155.6 \text{ lb}$$

$$P = 88.5 \text{ lb} \quad \Leftarrow \textbf{Ans.}$$

Substituting in Equation (d), we obtain:

$$Q = 108.5 \text{ lb} \quad \Leftarrow \textbf{Ans.}$$

EXAMPLE 1–17

In solving a three-dimensional equilibrium problem, the following equations are obtained:

$$0.429F_1 - 0.549F_2 - 0.570F_3 = 0 \quad \text{(a)}$$

$$0.857F_1 - 0.824F_2 + 0.684F_3 = 400 \text{ N} \quad \text{(b)}$$

$$0.286F_1 + 0.1374F_2 - 0.456F_3 = 0 \quad \text{(c)}$$

Solve the equations for the three unknown forces: F_1, F_2, and F_3.

Solution. Note that all three equations contain the three unknowns. We must eliminate one of the three unknowns (say, F_3) from the three equations. First, the coefficients of F_3 in the three equations are reduced to either +1 or −1 as follows:

$$\text{(a)} \div 0.570: \quad 0.753F_1 - 0.963F_2 - F_3 = 0 \quad \text{(d)}$$

$$\text{(b)} \div 0.684: \quad 1.253F_1 - 1.205F_2 + F_3 = 585 \text{ N} \quad \text{(e)}$$

$$\text{(c)} \div 0.456: \quad 0.627F_1 + 0.301F_2 - F_3 = 0 \quad \text{(f)}$$

The unknown F_3 can be eliminated by addition or subtraction:

$$\text{(d)} + \text{(e)}: \quad 2.006F_1 - 2.168F_2 = 585 \text{ N} \quad \text{(g)}$$

$$\text{(d)} - \text{(f)}: \quad 0.126F_1 - 1.264F_2 = 0 \quad \text{(h)}$$

The unknown F_2 can be eliminated from the above two equations by:

$$\text{(g)} \div 2.168: \quad 0.9253F_1 - F_2 = 269.8 \text{ N} \quad \text{(i)}$$

$$\text{(h)} \div 1.264: \quad 0.0997F_1 - F_2 = 0 \quad \text{(j)}$$

$$\text{(i)} - \text{(j)}: \quad 0.8256F_1 = 269.8 \text{ N} \quad \text{(k)}$$

From Equation (k), we get:

$$F_1 = 327 \text{ N}$$ ⇐ **Ans.**

Substituting in Equation (j) yields:

$$F_2 = 32.6 \text{ N}$$ ⇐ **Ans.**

Substituting in Equation (d) yields:

$$F_3 = 215 \text{ N}$$ ⇐ **Ans.**

Cramer's Rule for Two Equations. The solution to two linear equations containing two unknowns can be expressed in terms of *determinants of the second order,* defined as follows:

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 \qquad (1\text{–}11)$$

Equation 1–11 is a square array of numbers with two rows and two columns, enclosed by vertical bars. Each number in the array is called an *element.* Elements a_1 and b_1 are in the *first row*; elements a_2 and b_3 are in the *second row*; elements a_1 and a_2 are in the *first column*; elements b_1 and b_2 are in the second column. The *principal diagonal* is the direction along elements a_1 and b_2. The *secondary diagonal* is the direction along elements a_2 and b_1. The value of a determinant of the second order is *the product of the two elements along the principal diagonal subtracting the product of the two elements along the secondary diagonal.*

Using second-order determinants, the solution to a system of two linear equations with two unknowns in the form

$$a_1x + b_1y = k_1 \qquad (1\text{–}12)$$

$$a_2x + b_2y = k_2 \qquad (1\text{–}13)$$

can be written as

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad (1\text{–}14)$$

where

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \qquad D_x = \begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix} \qquad D_y = \begin{vmatrix} a_1 & k_1 \\ a_2 & k_2 \end{vmatrix} \qquad (1\text{–}15)$$

In Equation 1–15, the elements of determinant D are made up of the coefficients of x and y in the two equations. The elements in determinant D_x are essentially the same as those of determinant D, except that the elements in the first column (the coefficients of x) are replaced by the right-hand constant k's. Similarly, the elements in determinant D_y are essentially the same as those of determinant D, except that the elements in the second column (the coefficients of y) are replaced by the right-hand constant k's. The solution given in Equations 1–14 and 1–15 is known as *Cramer's rule.*

EXAMPLE 1–18

Solve the following equations for x and y by using Cramer's rule.

$$4x + y = 10$$

$$3x - 5y = 19$$

Solution. First, we set up and evaluate determinant D using the coefficients of x and y as elements:

$$D = \begin{vmatrix} 4 & 1 \\ 3 & -5 \end{vmatrix} = 4(-5) - 3(1) = -23$$

Then set up D_x by replacing the elements in the first column of D (the coefficients of x) with the right-hand constants:

$$D_x = \begin{vmatrix} 10 & 1 \\ 19 & -5 \end{vmatrix} = 10(-5) - 19(1) = -69$$

The determinant D_y can be set up by replacing the elements in the second column of D (the coefficients of y) with the right-hand constants:

$$D_y = \begin{vmatrix} 4 & 10 \\ 3 & 19 \end{vmatrix} = 4(19) - 3(10) = 46$$

By Cramer's rule, the solution is

$$x = \frac{D_x}{D} = \frac{-69}{-23} = 3 \qquad \Leftarrow \textbf{Ans.}$$

$$y = \frac{D_y}{D} = \frac{46}{-23} = -2 \qquad \Leftarrow \textbf{Ans.}$$

Cramer's Rule for Three Equations. Cramer's rule can also be applied to three linear equations with three unknowns in the form

$$a_1x + b_1y + c_1z = k_1 \qquad (1\text{–}16)$$

$$a_2x + b_2y + c_2z = k_2 \qquad (1\text{–}17)$$

$$a_3x + b_3y + c_3z = k_3 \qquad (1\text{–}18)$$

The solution to the equations is

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D} \qquad (1\text{–}19)$$

where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad (1\text{–}20)$$

$$D_x = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix} \qquad (1\text{–}21)$$

$$D_y = \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix} \qquad (1\text{–}22)$$

$$D_z = \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix} \qquad (1\text{–}23)$$

These determinants, having three rows and three columns, are called *determinants of the third order.* In Equation 1–20, the elements of the determinant D are made up of the coefficients of x, y, and z in the three equations. In Equation 1–21, the elements in the determinant D_x are essentially the same as those of the determinant D, except that the elements in the first column (the coefficients of x) are replaced by the right-hand constant k's.

Similarly, in Equation 1–22, the elements in the determinant D_y are essentially the same as those of determinant D, except that the elements in the second column (the coefficients of y) are replaced by the right-hand constant k's. And in Equation 1–23, the elements in the determinant D_z are essentially the same as those of determinant D, except that the elements in the third column (the coefficients of z) are replaced by the right-hand constant k's.

The value of a third-order determinant can be computed from

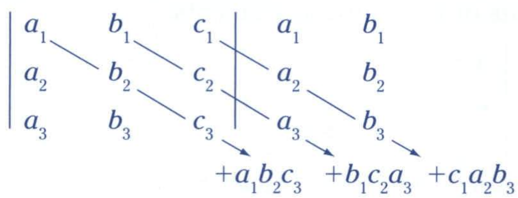
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3 \quad (1\text{–}24)$$

The computation of the right-hand side of Equation 1–24 can be simplified by using the following steps:

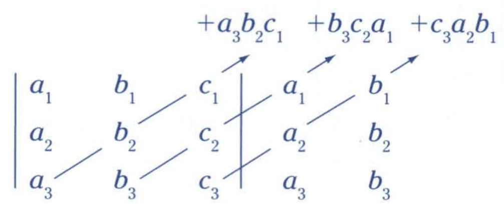
1. Duplicate the first and the second columns and place them to the right of the determinant, as shown in the following:

$$\left|\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}\right| \begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}$$

2. Find the product of the three elements along the principal diagonal, and the products of the three elements along the parallel diagonals to the right. The sum of the three products gives the first three terms of the expression in Equation 1–24, as shown in the following:



3. Find the product of the three elements along the secondary diagonal, and the products of the three elements along the parallel diagonals to the right, as shown in the following. Subtracting these three products from the previous sum gives the value of the determinant.



EXAMPLE 1–19

Find the value of the following third-order determinant:

$$\begin{vmatrix} 4 & -3 & 2 \\ 0 & -2 & 4 \\ 2 & 0 & -5 \end{vmatrix}$$

Solution. Following the steps described above, we find

$$\begin{array}{ccc|cc} + & + & + & - & - \; - \\ 4 & -3 & 2 & 4 & -3 \\ 0 & -2 & 4 & 0 & -2 \\ 2 & 0 & -5 & 2 & 0 \end{array}$$

$$= (4)(-2)(-5) + (-3)(4)(2) + 0 - (2)(-2)(2) - 0 - 0$$

$$= 24 \qquad \Leftarrow \textbf{Ans.}$$

EXAMPLE 1–20

Solve the following equations for x, y, and z by using Cramer's rule.

$$3x \quad + z = 0 \qquad \text{(a)}$$

$$2x - y + 4z = 8 \qquad \text{(b)}$$

$$4x - 3y + z = -7 \qquad \text{(c)}$$

Solution. First, we set up and evaluate determinant D using the coefficients of x, y, and z as elements:

$$D = \left|\begin{array}{ccc|cc} 3 & 0 & 1 & 3 & 0 \\ 2 & -1 & 4 & 2 & -1 \\ 4 & -3 & 1 & 4 & -3 \end{array}\right.$$

$$= (3)(-1)(1) + 0 + (1)(2)(-3) - (4)(-1)(1) - (-3)(4)(3) - 0$$

$$= 31$$

Replacing the elements of the first column of D (the coefficient of x) with the right-hand constants, we get

$$D_x = \left|\begin{array}{ccc|cc} 0 & 0 & 1 & 0 & 0 \\ 8 & -1 & 4 & 8 & -1 \\ -7 & -3 & 1 & -7 & -3 \end{array}\right.$$

$$= 0 + 0 + (1)(8)(-3) - (-7)(-1)(1) - 0 - 0$$

$$= -31$$

By Cramer's rule, we find

$$x = \frac{D_x}{D} = \frac{-31}{31} = -1 \qquad \Leftarrow \textbf{Ans.}$$

Substituting into Equation (a), we find

$$z = -3x = -3(-1) = 3 \qquad \Leftarrow \textbf{Ans.}$$

Substituting into Equation (b), we find

$$y = 2x + 4z - 8$$
$$= 2(-1) + 4(3) - 8 = 2 \qquad \Leftarrow \textbf{Ans.}$$

1-13
GENERAL PROCEDURE FOR PROBLEM SOLUTION

Extensive applications of statics and strength of materials are based on a few simple principles. The most effective way of learning this subject is to solve problems of different levels of complexity. The following general procedure is helpful:

1. Read the problem carefully. Identify the given data and the unknown quantities to be determined.
2. Make a neat sketch showing all the quantities involved. For some problems, it may be helpful to tabulate the given data and the computed results.
3. Apply the relevant principles and express the physical conditions in mathematical form. The solution must be based on the principles and theorems presented in the text and must be executed in a logical manner.
4. The equations obtained must be dimensionally homogeneous. Values in consistent units must be used for substitution. The answer obtained must be rounded off to the proper degree of accuracy or precision.
5. Use your common sense and judgment to determine if the answer obtained is reasonable. In some problems, there are conditions in which answers can be checked. If such conditions are available, always use them to check the answers.
6. The engineering profession requires work that meets high standards. Students preparing to enter an engineering career must present their work in a neat and organized fashion.

1-14
SUMMARY

Forces. *Mechanics* is a physical science that studies the effects of forces. Forces are vector quantities. *Vector quantities* are characterized by a magnitude, a point of application, and a direction.

Types of Forces. Forces can be applied on a body by *direct contact* or through *remote action.* Forces can be *concentrated* at a point or *distributed* along a length, over an area, or throughout the entire body. *External forces* are exerted on the body by another body. *Internal forces* are the resisting forces within a body.

Types of Force Systems. Force systems can be classified into the following three types, depending on whether they are coplanar or spatial, concurrent or nonconcurrent.

1. Concurrent-coplanar force system
2. Nonconcurrent-coplanar force system
3. Spatial force system

Newton's Three Laws. These three laws form the foundation for the study of *Newtonian mechanics.* The first law deals with conditions for equilibrium of a particle and thereby lays the foundation for the study of statics. The second law provides the basic formulation for the study of dynamics. The third law provides the basic understanding for the nature of action and reaction forces.

The Principle of Transmissibility. The point of application of a force may be placed anywhere along the line of action of the force without changing the external effects of the force. However, the line of action and the direction of a force must be well defined. For the internal effect or the deformation of a body, a force acting on the body must have a fixed point of application, and therefore the principle of superposition does not apply.

System of Units. Two systems of units are used in this book: the U.S. customary units and the SI units. The base units in the U.S. system are the foot, second, and pound. The base unit for force (or weight), the pound, is dependent on gravitational attraction; it is therefore a *gravitational system.* The base units in the SI system are the meter, second, and kilogram. The base unit for mass, the kilogram, is independent of gravitational attraction; it is therefore an *absolute system.*

Rules for Numerical Computations. Calculated results should always be rounded off according to the following rules:

Rule 1 When approximate numbers are *multiplied* or *divided,* the result is expressed to the *same accuracy* as the least accurate number.

Rule 2 When approximate numbers are *added* or *subtracted,* the result is expressed to the *same precision* as the least precise number.

Mathematics Used in Mechanics. Some fundamental mathematical skills are required of the student. For example, students must be able to perform elementary algebraic manipulations, solve a right triangle using the Pythagorean theorem and trigonometric functions, solve an oblique triangle using the law of sines and/or the law of cosines, and solve two or three simultaneous linear equations.

General Procedure for Problem Solution. Problems must be solved in a logical and orderly manner. Students must learn to analyze the problem carefully. Make necessary sketches and apply the relevant principles. Equations must be solved by using proper mathematical operations. Results should be checked against certain required conditions or judged to be reasonable using common sense. Work must be presented in a neat and organized fashion.

PROBLEMS

Section 1–1 Introduction to Mechanics

1–1 (*a*) What are the characteristics of a rigid body?
(*b*) In the study of statics and dynamics, bodies are considered rigid. Why?
(*c*) In the study of strength of materials, why is it important to consider the bodies deformable?

1–2 Identify whether each of the following is a topic in statics, dynamics, or strength of materials.
(*a*) Determining the size of a beam
(*b*) Calculating the reactions on a ladder
(*c*) Studying the motion of a projectile
(*d*) Calculating the deflections of a beam
(*e*) Determining the forces in truss members
(*f*) Studying the motion of a pendulum

Section 1–3 Scalar and Vector Quantities

1–3 What are the characteristics of a vector quantity?

1–4 In each of the following, indicate whether it is a scalar or a vector quantity.
(*a*) 60 minutes
(*b*) A displacement of 300 feet due east
(*c*) An upward force of 5 kN
(*d*) $1000.00
(*e*) A downward gravitational acceleration of 9.81 m/s^2
(*f*) A 50-kg mass

Section 1–5 Types of Force Systems

1–5 Name the force system in which all the forces are on a single plane and passing through a common point.

1–6 Name the force system in which the spatial forces do not meet at a common point.

Section 1–6 Newton's Laws

1–7 What is the meaning and significance of the mass of a body?

1–8 What is the significance of Newton's third law?

Section 1–7 The Principle of Transmissibility

1–9 Do we have to define the point of application of a force all the time?

1–10 What are the conditions when the principle of transmissibility can or cannot be applied?

Section 1–8 Systems of Units

1–11 What are the differences between the gravitational system of units and the absolute system of units?

1–12 What is the weight of a 5-slug mass in pounds?

1–13 What is the mass in slugs of a body weighing 500 lb?

1–14 What is the weight in newtons of a 10-kg mass?

1–15 What is the mass in kilograms of a body weighing 1000 N?

1–16 What is the weight of a 10-Mg mass in kilonewtons (kN)?

1–17 What is the mass in kilograms of a body weighing 100 N?

1–18 An astronaut weighs 150 lb on the surface of the earth. Determine (*a*) the mass of the astronaut in slugs, and (*b*) his weight in pounds on the moon, where the gravitational acceleration is 5.30 ft/s^2. What is his mass on the moon?

1–19 Reduce the following SI units to the units indicated.
(*a*) 6.38 Gg to kg
(*b*) 900 km to m
(*c*) 3.76×10^7 g to Mg
(*d*) 70 mm to m
(*e*) 23 400 N to kN

Section 1–9 Unit Conversion

1–20 A car travels at 60 mph. What is the equivalent speed in ft/min?

1–21 The world record for the men's 100-m dash is 9.82 s. What is the equivalent speed in mph?

1–22 The specific weight (weight per unit volume) of concrete is 150 lb/ft^3. What is its equivalent value in kN/m^3?

1–23 The mean radius of the earth is 6371 km. Determine its equivalent value in miles.

1–24 Use the conversion factors listed in Table 1–2 to convert the following SI units into the U.S. customary units indicated.
(*a*) 9.81 m/s^2 to ft/s^2
(*b*) 100 MN/m^2 to ksi (kips/in.^{2})
(*c*) 10 m/s to mph

1–25 Use the conversion factors listed in Table 1–2 to convert the following U.S. customary units into the SI units indicated.
(*a*) 200 lb-ft to N · m
(*b*) 600 mph to km/h
(*c*) 100 hp to kW

Section 1–10 Consistency of Units in an Equation

Section 1–11 Rules for Numerical Computations

In Problems 1–26 to 1–36, evaluate the given formula for the quantity indicated. Round off the results to a proper number of significant digits.

1–26 A circular area can be computed by the formula $A = \pi r^2$. Find the area of a circle having a radius of 3.25 ft.

1–27 Use the formula $A = \pi r^2$ for circular areas to find the area of a circular lot of 268 ft diameter in acres (1 acre = 43 560 ft^2).

1–28 The vertical distance y traveled by a freely falling body can be computed from the formula

$$y = v_0 t + \frac{1}{2} g t^2$$

where v_0 is the initial velocity, g is the gravitational acceleration, and t is the time of falling. Find the distance traveled by a falling body with an initial downward velocity of 2.25 m/s for 30 s.

1–29 Use the equation in Problem 1–28 to find the distance traveled by a body falling with an initial downward velocity of 25.0 ft/s for 15.0 s.

1–30 Use the formula in Problem 1–28 to find the time it takes a body to fall a vertical distance 1250 ft starting from rest.

1–31 The force F in a linear spring is given by $F = kx$, where k is the spring constant (force per unit length of spring deflection) and x is the spring deflection. Find the force in a spring with a spring constant of 100 lb/ft and a deflection of 3.00 in.

1–32 The frequency f in Hz (cycles per second) of an oscillating body can be computed from

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where k is the spring constant and m is the mass of the body. Find the frequency of vibration of a 30.5-kg mass supported on a spring with a spring constant of 1.57 kN/m.

1–33 Using the formula in Example 1–5, find the elongation in a steel wire of 2.00-mm diameter and a length of 10.0 m that is subjected to an axial force of 400 N. For steel, $E = 270\ \text{GN/m}^2$.

1–34 An object falling from rest through a height h reaches a velocity $v = \sqrt{2gh}$, where g is the gravitational acceleration. If a rock falls from a cliff 125 ft above the ground, what is its velocity when it hits the ground?

1–35 Using the formula in Problem 1–34, find the velocity of a rock after falling 40 m from a cliff.

1–36 The area of a triangle with three given sides a, b, and c can be computed from the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = (a + b + c)/2$. Find the area of a triangle with sides 5.45 ft, 6.85 ft, and 7.39 ft.

Section 1–12 A Brief Review of Mathematics

1–37 The hypotenuse of a right triangle is 700 mm and one of the acute angles is 35°. Find the lengths of the other two sides.

1–38 The hypotenuse and one side of a right triangle are 15 in. and 10 in., respectively. Determine the angle between the hypotenuse and the shorter side.

1–39 Determine the distance between two points BC across the river shown in Fig. P1–39 if the angle at C is laid out at an angle of 90°, the distance CA is laid out 400 ft away, and angle A is measured to be 49.5°.

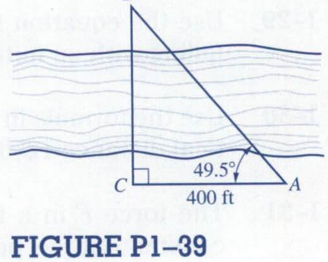


FIGURE P1–39

1–40 Find the angle between the wings of the toggle bolt shown in Fig. P1.40.

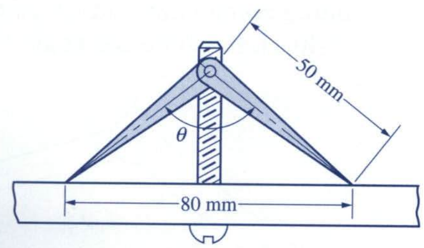


FIGURE P1–40

1–41 The flagpole in Fig. P1.41 has two sections, *AB* and *BC*. The angles α and β, measured at *D* at a distance of 50 m from the pole, are 40° and 30°, respectively. Find the heights *a* and *b* of the two sections of the pole.

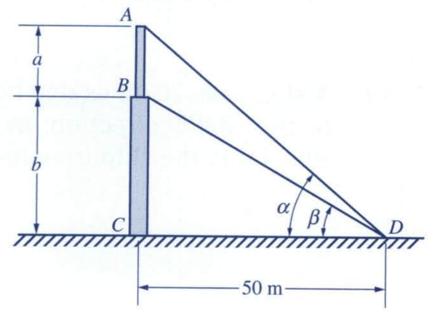


FIGURE P1–41

1–42 Determine the height *h* and the lengths *a* and *b* of the roof truss in Fig. P1–42. (*Hint:* Draw *AD* perpendicular to *BC*; *AD* bisects both *BC* and *EF*. Solve *AD* = *h* from the right triangle *ABD*, and *ED* = *b*/2 from the right triangle *AED*.)

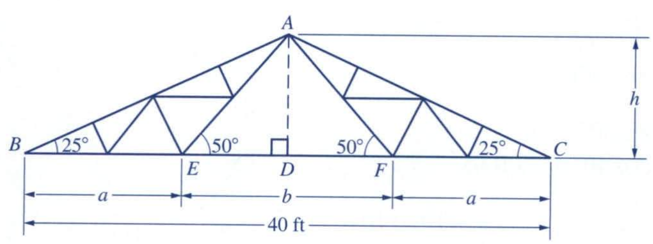


FIGURE P1–42

In Problems 1–43 to 1–47, find the unknown elements of an oblique triangle if three elements are given. See Fig. 1–7 for the notations used.

1–43 $a = 100$ mm, $A = 35°$, $B = 65°$

1–44 $a = 3.5$ ft, $B = 32°$, $C = 105°$

1–45 $b = 12$ m, $c = 15$ m, $A = 45°$

1–46 $a = 9$ in., $b = 10$ in., $C = 120°$

1–47 $a = 2.3$ m, $b = 4.5$ m, $c = 5.4$ m

1–48 The reciprocal engine in Fig. P1–48 consists of a crankshaft *OA* 100 mm long and a connecting rod AB 250 mm long. In the crankshaft position shown, $\alpha = 40°$. Determine the angle β and the distance OB.

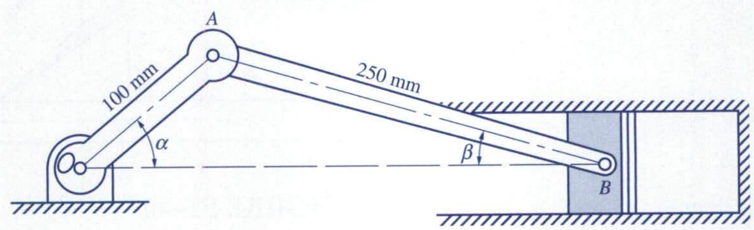


FIGURE P1–48

1–49 A ship sails 70 miles due north and then 90 miles in the N60°E direction, as shown in Fig. P1–49. How far is the ship from its starting point O?

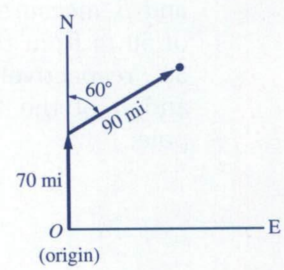


FIGURE P1–49

In Problems 1–50 to 1–54, solve the given system of linear equations by addition or subtraction.

1–50
$$3x + 5y = -8$$
$$5x - 3y = 15$$

1–51
$$3.45x - 2.65y = 2.77$$
$$1.86x + 3.76y = 9.85$$

1–52
$$T\sin 10° - P\sin 40° = 0$$
$$T\cos 10° - P\cos 40° = 200 \text{ lb}$$

1–53
$$-0.429P + 0.231Q = 1920 \text{ lb}$$
$$-0.857P - 0.923Q - 0.923R = 2880 \text{ lb}$$
$$0.286P - 0.308Q - 0.385R = 2160 \text{ lb}$$

1–54
$$-0.444x - 0.857y + 0.667z = 0$$
$$0.444x + 0.429y + 0.667z = 17 \text{ kN}$$
$$0.778x - 0.286y - 0.333z = 0$$

1–55 Solve the equations in Problem 1–50 by Cramer's rule.

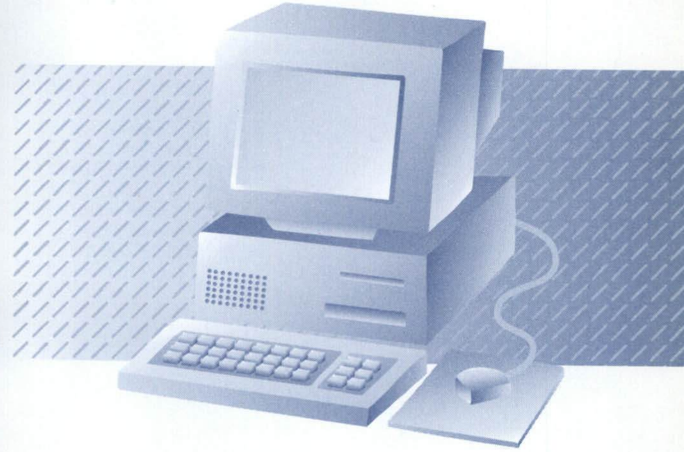
1–56 Solve the equations in Problem 1–51 by Cramer's rule.

1–57 Solve the equations in Problem 1–52 by Cramer's rule.

1–58 Solve the equations in Problem 1–53 by Cramer's rule.

1–59 Solve the equations in Problem 1–54 by Cramer's rule.

Computer Program Assignments



For each of the following problems, write a computer program using an appropriate programming language with which you are most familiar. Make the program user friendly by incorporating plenty of comments and input prompts so that the user will understand the input data to be entered and the limitations of their values. The output should include the data entered and the computed results, and they must be well labeled to identify each quantity.

C1–1 Write a computer program that can be used to solve a system of two equations with two unknowns using Cramer's rule. The user input should be the coefficients of x and y, and the right-hand constant of each equation. The output must include the given equations, the value of the determinants, and the solution to the equations. If D, D_x, and D_y are all equal to zero, the two equations are dependent and there is no unique solution. If D is zero, but either one or both D_x and D_y are not zero, the two equations are inconsistent and there is no solution. Use this program to solve (*a*) Example 1–16, (*b*) Problem 1–51, and (*c*) Problem 1–52.

C1–2 Modify the program written for Problem C1–1 so that it can be used to solve a system of three equations with three unknowns using Cramer's rule. Use this program to solve (*a*) Example 1–20, (*b*) Problem 1–53, and (*c*) Problem 1–54.